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Observation of a superposition of orthogonally polarized geometric beams with a *c*-cut Nd:YVO_4 crystal

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Abstract We investigate a superposition of orthogonally polarized geometric beams generated from a *c*-cut Nd:YVO_4 laser. Experimental results reveal that a geometric mode possesses a linearly polarized state and circularly polarized states in opposite directions at the same time and the superposition of orthogonally polarized geometric beams can be generated systematically by controlling the pump offset. We use the birefringence theory to analyze the data, and the numerical results have a good agreement with the experimental results.

1 Introduction

Polarization is one of the most important properties of light. The vector nature of light with polarization has been extensively explored in optical vortex beams [1, 2], focus shaping [3, 4], optical trapping [5], quantum information [6], and laser applications [7, 8]. The potential applications of polarized light make it important to study the polarized beam-matter interaction [9–11] and to generate optical beams with specific polarization states such as cylindrical vector beams [12]. Recently, light beams possessing space-variant polarization states have attracted much attention in laser fields. The birefringent crystal was used to generate radially polarized laser beam in a hemispherical cavity [7], and an anisotropic laser media was used to form the structured singularities in a laser cavity [13, 14]. Different selections of the cavity skeleton and medium lead to different polarization properties of light. Cylindrically polarized Laguerre-Gaussian beams can be created by lasers with intracavity Brewster windows [15]. Theoretical studies also provide the vector beam solution for a description of the free-space modes excited by light exiting a fiber [16]. A great deal of effort has been made on the polarization states related to the eigenmodes and the circular symmetry in optical systems. What seems to be lacking, however, is the formation of geometric modes possessing space-variant polarization states in degenerate laser cavities.

Analysis of structured beams generated from degenerate laser cavities has revealed the importance of the relation between eigenmodes and structured beams which result from the superposition of eigenmodes [17, 18]. The geometric beams are the result of phase and frequency locking of degenerate Hermite–Gaussian modes, and resemble closed ray paths inside a cavity. Three-dimensional coherent states localized on classical parametric surfaces and two-dimensional coherent states localized on geometric trajectories are ubiquitous in an *a*-cut Nd:YVO_4 laser, and these optical fields are linearly polarized along the *c*-axis due to the large stimulated emission cross section [18–20]. Furthermore, studying the formation of geometric modes from degenerate cavities paves the way for us to explore the space-variant polarization states of geometric beams. In this paper, we propose a superposition of orthogonally polarized states embedded in geometric beams by use of a birefringent laser crystal, and the self-organized formation of geometric beams with space-variant polarization states is manifested experimentally. To our knowledge, it is the first observation of multi-polarized states (two opposite circularly polarized beams and one linearly polarized beam) generated from a laser cavity.

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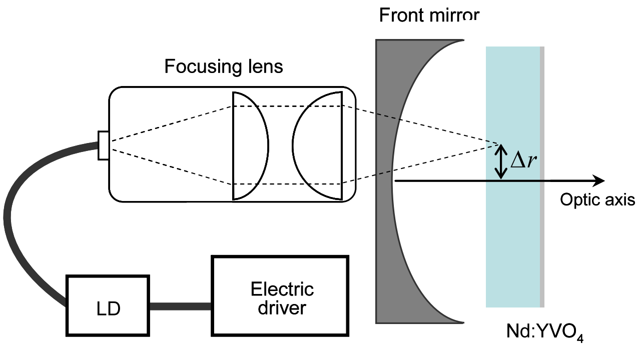

Fig. 1 Diagram of experimental setup

2 Experimental setup

The laser system is a diode-pumped Nd:YVO_4 laser. Figure 1 shows a schematic diagram of the laser cavity arrangement. A concave mirror with the radius of curvature $R = 15$ mm is used as a front mirror and its reflectivity is 99.8% at 1064 nm. The cavity length is set as $L = 3R/4$ to provide a degenerate cavity, and the ratio between transverse and longitudinal mode spacing is given by 1/3 [19]. The laser gain medium is a c-cut 2.0-at% Nd:YVO_4 crystal with a length of 1 mm. The optic axis of the Nd:YVO_4 crystal is parallel to the cavity axis. One side of the Nd:YVO_4 crystal is coated for antireflection at 1064 nm; the other side is coated to be an output coupler with a reflectivity of 99%. The pump source is an 809 nm fiber-coupled laser diode with a core diameter of 100 μm, a numerical aperture of 0.16, and a maximum power of 3 W. A focusing lens with 20 mm focal length and 90% coupling efficiency is used to reimage the pump beam into the laser crystal, and it is mounted on a two-dimensional mechanical stage to adjust the pump offset Δr. The Nd:YVO_4 was a positive birefringent crystal with $n_o = 1.9573$ and $n_e = 2.1652$ at 1064 nm, respectively. Since the YVO_4 crystal belongs to the group of oxide compounds crystallizing in a zircon structure with tetragonal space group, the Nd-doped YVO_4 crystals show strong polarization-dependent fluorescence emission due to the anisotropic crystal field. When a laser beam enters a c-cut Nd:YVO_4 crystal with an angle parallel to the optic axis, the birefringence will be zero because of high-level transverse isotropy. On the other hand, when a laser beam passes a c-cut Nd:YVO_4 crystal with an angle θ to the optic axis, the polarization state changes according to the phase retardation which resulted from the birefringence. It is worthy to note that a c-cut Nd:YVO_4 crystal is different from a conventional Nd:YVO_4 crystal which is cut along the a axis to use the largest stimulated emission cross section for lowering the lasing threshold.

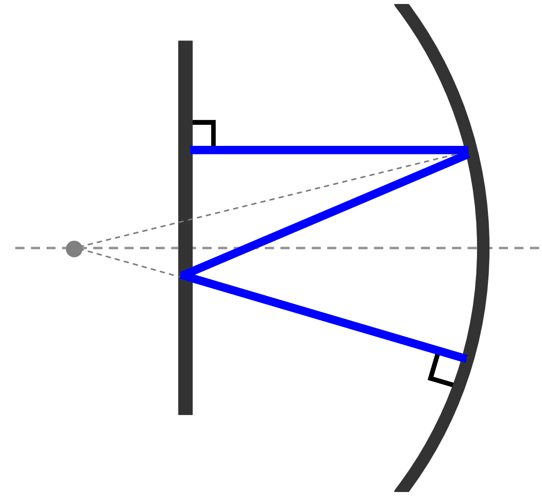

Fig. 2 A closed periodic orbit of N mode in the degenerate cavity

3 Experimental results and physical explanation of a superposition of orthogonally polarized geometric beams

The analysis of lasing modes localized on the geometric trajectory provides a starting-point [21]. When the optical cavity is a degenerate cavity, the dominating mode is not the pure Hermite–Gaussian mode but the mode that can be viewed as multi-bounce Gaussian beams traveling in closed off-axis trajectories. On the other hand, the superposition of degenerate Hermite–Gaussian modes forms a stationary coherent wave which corresponds to the relation between wave optics and geometric optics [17, 18]. When the lasing mode with a geometric trajectory is generated from an off-axis pumped laser system in a degenerate cavity length, the directions of the beams emitted from the cavity can be clearly clarified. Figure 2 shows a geometric beam of the N mode in the degenerate cavity ($L = 3R/4$) in accordance with the reflection law. The N mode can be regarded as off-axis folded Gaussian beams that go through six reflections in completing a round trip in the cavity. Furthermore, the refractive indices are indicated in Fig. 3b to illustrate the polarization states. As the laser beam parallel to the optic axis starts from the upper position in the laser medium to travel backward to the front mirror, there is no phase difference between the two transverse electric fields due to the same refractive index indicated by the arrow and arrow nock. Consequently, the beam parallel to the optic axis is of a linearly polarized state. The beam is reflected consecutively by the front mirror and travel forward to pass through the laser medium with an angle θ to the optic axis. The transverse electric fields related to the two refractive indices, n_o and n_{eff}, result in the different refractive angles, θ_o and θ_{eff}. The zoom-in illustration shown in Fig. 3b depicts the different paths of the two beams; however, the experimental lateral displacement

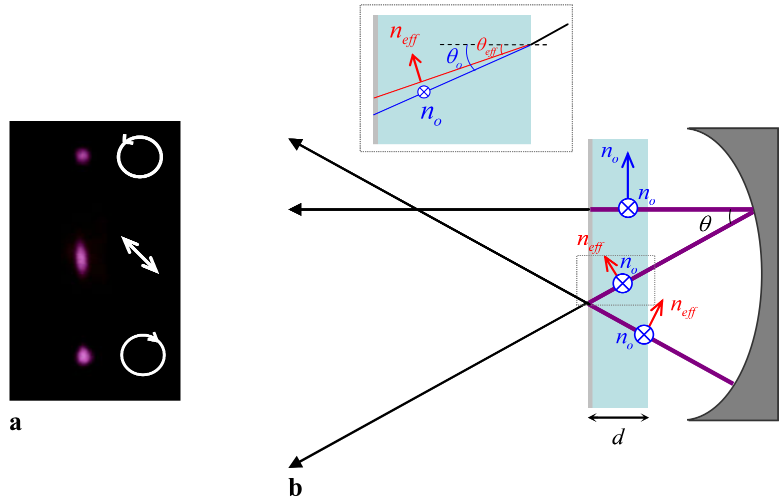

Fig. 3 (**a**) Experimental result of N mode in the far field. (**b**) Illustration of the N mode and the corresponding refractive indices with enlargement (*above left*)

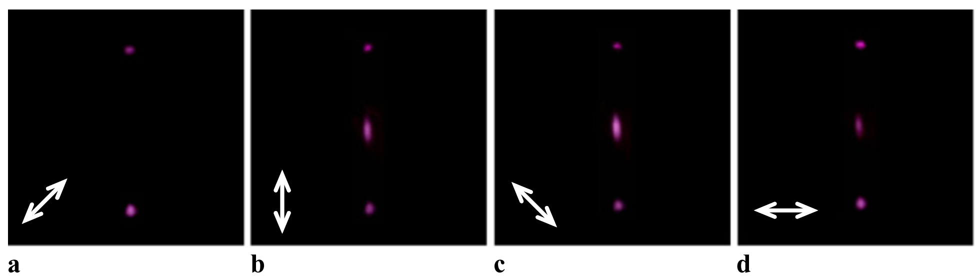

Fig. 4 Experimental polarization-resolved patterns according to the pattern in Fig. 2(b). (**a**) 45° polarization, (**b**) 90° polarization, (**c**) 135° polarization, (**d**) 180° polarization

Table 1 Conditions of the pump offset and related parameters for generation of the N modes

Δr (μm)	108	128	180	269	295	323	382
θ (degree)	3.82	4.94	5.85	6.62	7.32	7.95	8.53
m	2.99	4.99	6.99	8.94	10.92	12.87	14.80
Error (%)	0.33	0.20	0.14	0.67	0.73	1.00	1.33

of the two beams is small enough to maintain the localized N mode. The phase difference of the two transverse fields alters the polarization state of the beam. According to the theory of light propagation in a uniaxial crystal [22], the effective index n_{eff} is given by

$$n_{\mathrm{eff}} = \frac{n_o n_e}{\sqrt{n_e^2 \cos^2\theta + n_o^2 \sin^2\theta}}. \tag{1}$$

The difference of the two refractive indices leads to the phase retardation which can be represented as

$$\delta = \left(\frac{2\pi \cdot d}{\lambda}\right) \cdot \left(\frac{n_{\mathrm{eff}}}{\cos(\theta_{\mathrm{eff}})} - \frac{n_o}{\cos(\theta_o)}\right) \tag{2}$$

where d is the thickness of the laser medium and λ is the wavelength of the laser beam. When the phase retardation is an odd multiple of $\pi/2$, the polarization state of the laser beam which goes forward to pass the medium changes from linearly polarized to circularly polarized. For the geometry of the N mode, it is clear to see that a part of the laser beam which is reflected from the surface of the medium is reflected from the front mirror along the same trajectory to travel forward through the medium again. When the circularly polarized beam (the middle leg of the N mode inside the cavity) reflects from the surface of the medium and travel backward to the front mirror, the phase retardation of an odd multiple of $\pi/2$ is further added. Continuously, the lowest leg of the N mode inside the cavity reflects from the front mirror and travels forward to pass through the medium. The phase retardation of an odd multiple of $\pi/2$ is added again. Therefore, the difference of the phase retardation of the two beams from the lower legs of the N mode is an odd multiple of π. The polarization states of the two beams are circularly polarized in opposite directions. Figure 3a depicts the far-field experimental result. It is worthy to mention that the pumping beam is focused on the position near the intersection of the two legs of the N mode inside the gain medium to make the effective overlapping. The pump offset for the experimental result shown in Fig. 3a is 295 μm, the angle θ is 7.32°, and the phase retardation δ is about $11\pi/2$. A polarizer is used to check the polarization state of the lasing mode. Figure 4 shows that the lasing mode goes through the polarizer of different angles indicated below. Obviously, the laser beam in the middle is linearly polarized. We use a quarter-wave plate to double check the other two beams and make sure the polarization states. The far-field pattern of the N mode is demonstrated to be one linearly polarized beam on the center and two circularly polarized beams in opposite directions on both sides. Moreover, the larger the pump offset is, the larger angle between the two beams with opposite circularly polarized states is. It is important that the angle of the N mode dominates the polarization states of the geometric beam. To accomplish the quantitative analysis, we manipulate the pump offset to generate the geometric beams of the specific polarization states (two opposite circularly polarized beams and one linearly polarized beam) in discrete conditions. Table 1 depicts the off-axis conditions that the N modes with the superposition of orthogonally polarized states can be generated from the degenerate cavity. The phase retardation δ represents the phase difference of the transverse orthogonal electric fields when the beam passes the medium with an incident angle θ to the optic axis. For the case mentioned above, the phase retardation δ can be written as $m\pi/2$, and m is an odd number. So, it is clear to see that the experimental data of the phase retardation are in good agreement with the theoretical result. The error is within 1.5%. When the pump offset is smaller than 100 μm, the linearly polarized Hermite–Gaussian modes dominate in the cavity. Hence the experimental result shows that the least phase retardation to form the N mode in the laser cavity is $3\pi/2$. The superposition of orthogonally polarized states with geometric trajectory are generated systematically by manipulating the pump offset from 100 μm to 400 μm.

From the previous analyses, the explanation of the formation of the superposition of orthogonally polarized geometric beams is straightforward from the birefringence theory. At the same time, it is important to consider that the composition of the geometric mode is the superposition of Hermite–Gaussian modes. Recently, the study of optical modes helped us to realize that the theoretical analysis of optical modes plays an important role for us to understand the formation of various optical modes and the significance of optical modes [13, 14]. Polarization-independent optical modes can elegantly be analyzed with the coherent wave representation, because the distribution of the optical field is the only thing to be considered. Comparatively speaking, polarization-dependent optical modes are very complicated to be reconstructed by the coherent wave representation. So far, how to use wave representation to reconstruct the geometric beams with the superposition of orthogonally polarized states is beyond our understanding, and there is room for further investigation in the future.

4 Conclusion

In conclusion, we employ a simple laser system to demonstrate the superposition of orthogonally polarized geometric beams of N modes. The birefringent laser crystal leads to the orthogonally polarized beams with different optical paths and space-variant phase retardation inside the cavity. Each beam of the geometric mode possesses different polarization states: linearly polarized state and circularly polarized states with opposite directions. The numerical analysis for the angle is in good agreement with the experimental result. Moreover, the similar phenomenon also applies to other geometric modes generated from degenerate cavities which comprise a *c*-cut Nd:YVO_4 crystal as the laser crystal. The study of polarization states of geometric modes may provide some useful insights into applications.

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